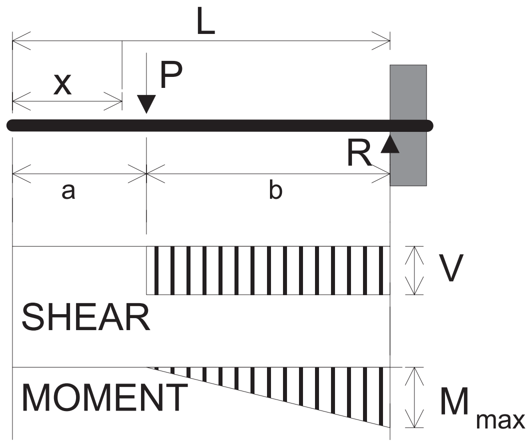


$R = V = P$

M_{max} (at fixed end) $= Pb$

M_x (when $x>a$) $= P(x-a)$

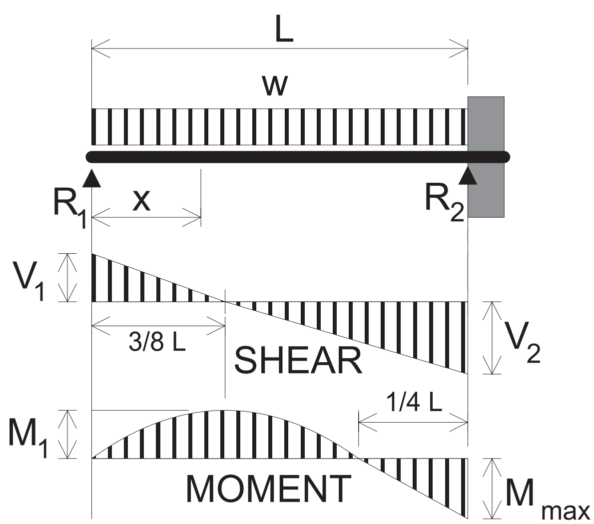
Δ_{max} (at free end) $= \dfrac{Pb^2}{6EI}(3L-b)$

Δ_a (at point of load) $= \dfrac{Pb^3}{3EI}$

Δ_x (when $x<a$) $= \dfrac{Pb^2}{6EI}(3L-3x-b)$

Δ_x (when $x>a$) $= \dfrac{P(L-x)^2}{6EI}(3b-L+x)$

Figure A.8 - Cantilever Beam - Concentrated Load at Any Point



$R_1 = V_1 = \dfrac{3wL}{8}$

$R_2 = V_2 = V_{max} = \dfrac{5wL}{8}$

$V_x = R_1 - wx$

$M_{max} = \dfrac{wL^2}{8}$

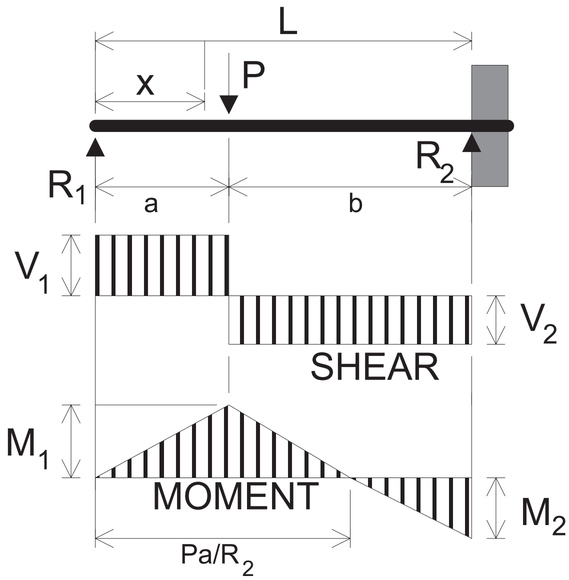
M_1 (at x =) $= \dfrac{3}{8}L = \dfrac{9}{128}wL^2$

$M_x = R_1x - \dfrac{wx^2}{2}$

Δ_{max} (at $x = \dfrac{L}{16}(1+\sqrt{33}) = 0.42L$) $= \dfrac{wL^4}{185EI}$

$\Delta_x = \dfrac{wx}{48EI}(L^3 - 3Lx^2 + 2x^3)$

Figure A.9 - Beam Fixed at One End, Supported at Other - Uniformly Distributed Load



$$R_1 = V_1 = \frac{Pb^2}{2L^3}(a + 2L)$$

$$R_2 = V_2 = \frac{Pa}{2L^3}(3L^2 - a^2)$$

$$M_1 \text{ (at point of load)} = R_1 a$$

$$M_2 \text{ (at fixed end)} = \frac{Pab}{2L^2}(a + L)$$

$$M_x \text{ (when } x<a) = R_1 x$$

$$M_x \text{ (when } x>a) = R_1 x - P(x-a)$$

$$\Delta_{max} \text{ (when } a<0.4L \text{ at } x = L\frac{L^2+a^2}{3L^2-a^2}) = \frac{Pa}{3EI}\frac{(L^2-a^2)^3}{(3L^2-a^2)^2}$$

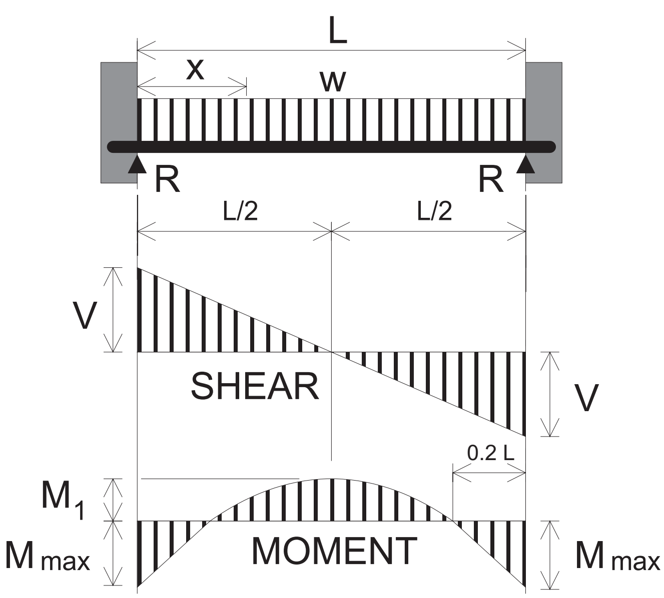
$$\Delta_{max} \text{ (when } a>0.4L \text{ at } x = L\sqrt{\frac{a}{2L+a}}) = \frac{Pab^2}{6EI}\sqrt{\frac{a}{2L+a}}$$

$$\Delta_a \text{ (at point of load)} = \frac{Pa^2b^3}{12EIL^3}(3L + a)$$

$$\Delta_x \text{ (when } x<a) = \frac{Pa^2x}{12EIL^3}(3aL^2 - 2Lx^2 - ax^2)$$

$$\Delta_x \text{ (when } x>a) = \frac{Pa}{12EIL^3}(L-x)^2(3L^2x - a^2x - 2a^2L)$$

Figure A.10 - Beam Fixed at One End, Supported at Other - Concentrated Load at Any Point



$$R = V = \frac{wL}{2}$$

$$V_x = w\left(\frac{L}{2} - x\right)$$

$$M_{max} \text{ (at ends)} = \frac{wL^2}{12}$$

$$M_1 \text{ (at center)} = \frac{wL^2}{24}$$

$$M_x = \frac{w}{12}(6Lx - L^2 - 6x^2)$$

$$\Delta_{max} \text{ (at center)} = \frac{wL^4}{384EI}$$

$$\Delta_x = \frac{wx^2}{24EI}(L - x)^2$$

Figure A.11 - Beam Fixed at Both Ends - Uniformly Distributed Loads